ON THE USE OF STATE-FEEDBACK IN THE DESIGN OF ELEMENTARY SWITCHING REGULATORS

Eduard Bertrán, Luis Martinez and Alberto Poveda
Escuela Técnica Superior de Ingenieros de Telecomunicación
Universidad Politécnica de Cataluña
Jordi Girona Salgado, s/n. Barcelona 08034. SPAIN

Abstract

The dc-to-dc buck switching converter is modelled by means of the State-Space Averaging Method [1] and techniques of Modern Control Theory are applied to obtain a class of state-feedback control laws. Three different strategies in the design of the control loop are then investigated. Namely, closed-loop pole assignment, optimal linear regulator and adaptive control. Predicting the transient response by means of simulation allows the comparison among the performances of the different strategies. The obtained results can be used in the optimum design of elementary switching regulators.

1.- Introduction

In the last years trends in dc-to-dc switching regulator control point at the increasing use of the current-programming mode. This technique constitutes a particular case of a two-loop state feedback control strategy in which the response of one of the loops is considerably faster than that of the other.

The work here reported investigates the use of state feedback in the design of elementary switching regulators. After modelling the dc-to-dc buck switching converter by means of the State-Space Averaging Method [1] in section 2, closed loop pole assignment is carried out in section 3. The use of an optimal linear regulator (OLR) is subsequently analyzed in section 4. In both strategies, the value of the inductor current is estimated by means of a Luenberger observer.

Finally, the use of adaptive control reducing additive noise in the output voltage is also presented.

2.- Converter Modelling

Figure 1 shows a dc-to-dc switching regulator whose power stage is a buck converter. Its corresponding block diagram is depicted in figure 2, where X is the converter state vector and k represents the feedback gain vector.

The converter small-signal behaviour can be modelled by means of the State-Space Averaging Method [1], which provides a single structure linear description equivalent to the periodically changing structure of the converter.

Since the converter operates in the continuous conduction mode its dynamic model can be obtained according to [1] from the following expression

$$\dot{\hat{x}} = A\hat{x} + B\hat{d} \qquad (1)$$

where $\hat{x}$ and $\hat{d}$ represents the perturbed values of state vector and duty-cycle respectively. They are related to their corresponding non-perturbed values as follows

$$d = D + \hat{d}$$

$$x = X + \hat{x}$$

being D and X the steady-state values of duty cycle and state vector respectively.

Also, matrices A and B are given by

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \qquad B = \begin{bmatrix} 1/L \\ 0 \end{bmatrix} \times E$$

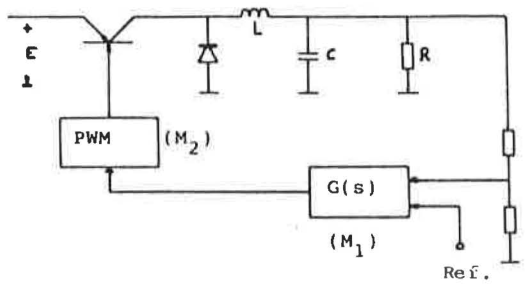

Fig. 1.- Buck switching regulator

Using the set of parameters L = 1,53mH , C = 310µF, R = 50Ω, E = 12v , f = 25KHz and D = 0,44, the following model is obtained

$$\dot{\hat{x}} = A\hat{x} + B\hat{d} \qquad (2)$$

where

$$A = \begin{bmatrix} 0 & -653,6 \\ 3225,8 & -64,51 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 653,6 \\ 0 \end{bmatrix} \times 12$$

which is the model that will be used to investigate the different control strategies in the next sections.

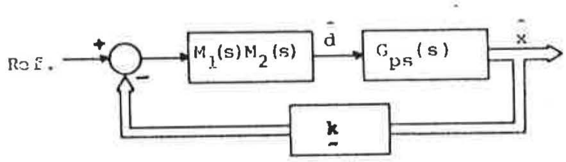

Fig. 2.- Small signal block diagram of the buck switching regulator

3.- Closed-Loop Pole Assignement

From the state space general formulation [3]

$$\dot{\hat{x}} = A\hat{x} + B\hat{d}$$
$$\hat{y} = C^T\hat{x} \qquad (3)$$

the control to output transfer funcion will be expressed as follows

$$G(s) = \frac{\hat{y}(s)}{\hat{d}(s)} = C^T \frac{Adj(sI - A)}{det(sI - A)} B \qquad (4)$$

where Adj means adjoint matrix, and I is the unit matrix

The system poles are determined from the equation

$$\det(sI - A) = 0 \tag{5}$$

In a closed-loop system such as that depicted in figure 3, equation (3) becomes

$$\hat{x} = (A - Bk)\hat{x} + B\hat{d} \tag{6}$$

and

$$G(s) = C^T(sI - A + Bk)^{-1}B \tag{7}$$

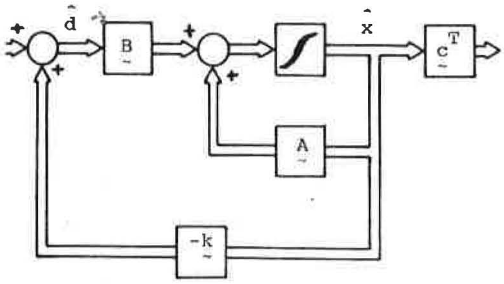

Fig. 3.- Block diagram of a state-feedback system

The corresponding closed-loop poles will be the roots of

$$\det(sI - A + Bk)^{-1} = 0 \tag{8}$$

Equation (8) in the case of a buck converter leads to

$$s^2 + s\left(\frac{k_1 E}{L} + \frac{1}{RC}\right) + \frac{1}{LC}(Ek_2 + 1) = 0 \tag{9}$$

where k_1 and k_2 are depicted in figure 4.

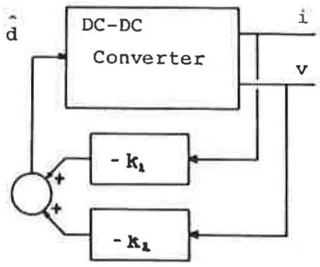

Fig. 4.- State feedback in the buck switching regulator

Equation (9) shows that closed pole assignement can be carried out by choosing the appropiate values of k_1 and k_2. In the case of the buck convertor, we have selected $k_1 = 0{,}502$ and $k_2 = 0{,}075$ which results in two real poles.

$$s_1 = s_2 = -2000$$

Figures 5 and 6 illustrate the simulated behaviour of the state variables of the previous system after introducing a step chage of 10% in the duty cycle.

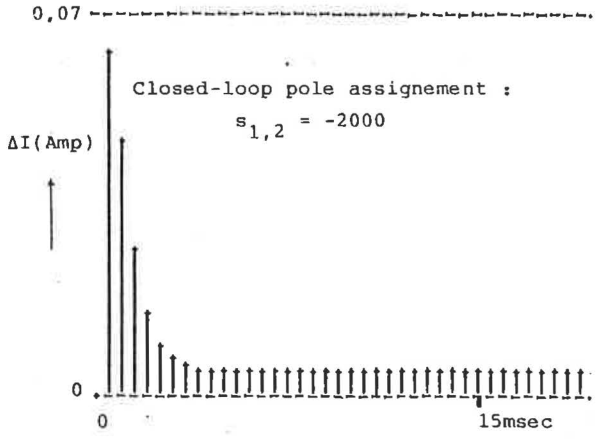

Fig. 5.- Response of the inductor current to a step change of 10% in the duty cycle

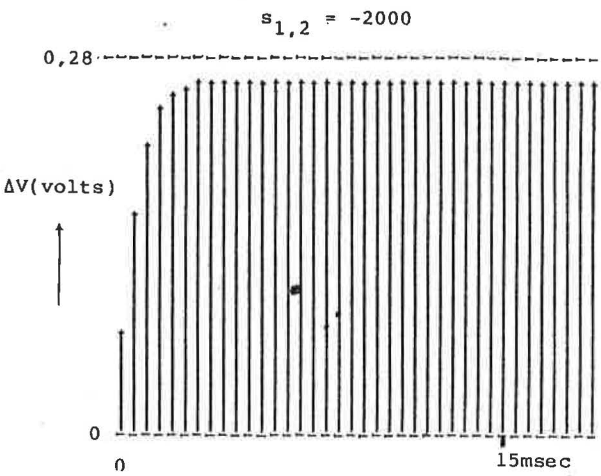

Fig. 6.- Response of the output voltage to a step change of 10% in the duty cycle.

4.- Optimal Linear Regulator (OLR)

The main objective in this case is to obtain the gain vector k so that a performance index may be minimized. This criterium will be expressed as

$$J = \int_0^\infty (\hat{x}^T Q \hat{x} + \hat{d}^t R d)\, dt \tag{10}$$

where Q and R are positive definite penalty matrices [3].

If the system is controllable, the control law minimizing J is

$$\hat{d} = -k\hat{x} = -R^{-1}B^T P\hat{x} \tag{11}$$

where P is the solution of the Riccati equation.

The success of a design based in the optimal linear regulator lies on the appropiate selection of both penalty matrix Q for the deviation of state vector and penalty matrix R for control effort.

Since the closed-loop system has no zeros in the origin of the s-plane, its response to input perturbations will exhibit a steady-state offset. To avoid this problem, a state-space enlarged with an integrative term [4] has been considered as shown in Figure 7.

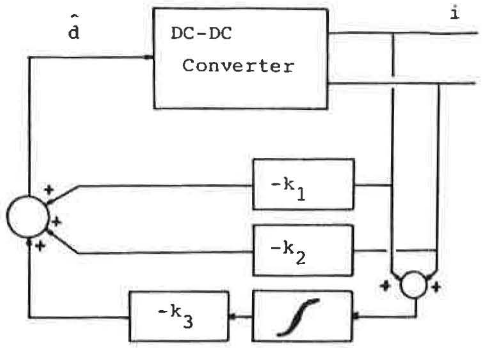

Fig. 7.- State-space modified with an integrative term

Therefore, in our case the enlarged state vector is

$$\hat{\underset{\sim}{x}}_1 = \begin{bmatrix} i \\ v \\ p \end{bmatrix} \qquad (12)$$

with

$$p = \int_{-\infty}^{0} (i(\lambda)+v(\lambda))d\lambda \qquad (13)$$

and the model described in equation (2) becomes

$$\dot{\hat{\underset{\sim}{x}}}_1 = \underset{\sim}{A}_1 \hat{\underset{\sim}{x}}_1 + \underset{\sim}{B}_1 \hat{d}$$

where

$$\underset{\sim}{A}_1 = \begin{bmatrix} 0 & -653,6 & 0 \\ 3225,8 & -64,51 & 0 \\ 1 & 1 & 0 \end{bmatrix} \qquad \underset{\sim}{B}_1 = \begin{bmatrix} 7843,2 \\ 0 \\ 0 \end{bmatrix}$$

Figures 8 and 9 show the simulation results using the performance index

$$J = \int_0^{\infty} (\hat{\underset{\sim}{x}}^T \underset{\sim}{Q} \underset{\sim}{x} + r\hat{d})dt$$

for different values of r. It must be noted that decreasing the value of r results in a lower steady-state offset.

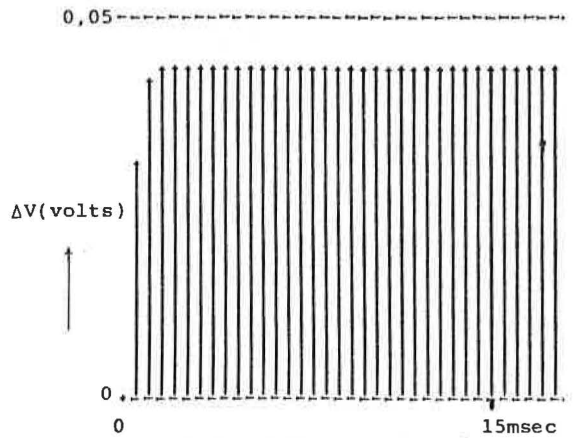

Fig.8.- Response of the output voltage to a step change of 10% in the duty cycle.OLR case,r=1, $\underset{\sim}{Q}$=diag(1,1,2)

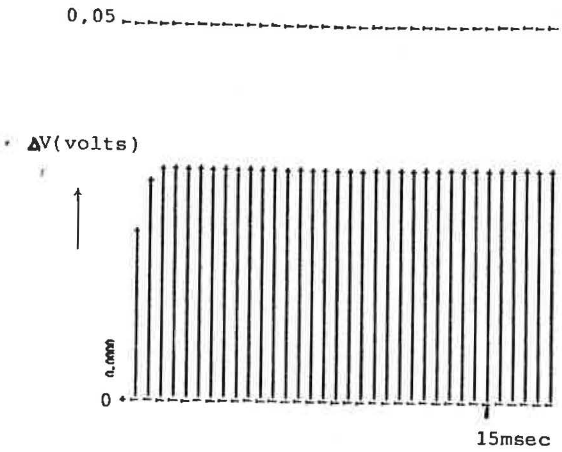

Fig. 9.- Response of the output voltage to a step change of 10% in the duty cycle, OLR case, r = 0,5, $\underset{\sim}{Q}$ = diag(1,1,2)

5.- Adaptive Control

From the block diagram based on parallel reference model [5] shown in figure 10.

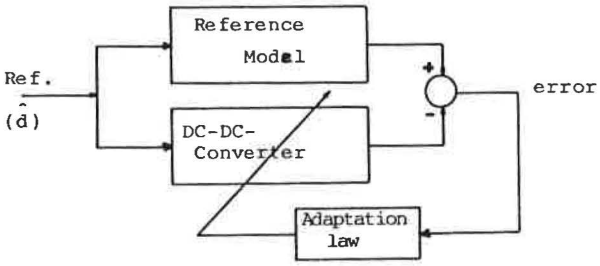

Fig. 10.- Parallel reference model

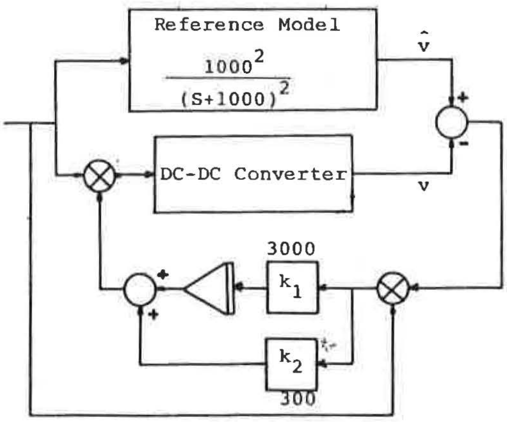

Fig. 11.- Adaptive control block diagram

Figure 12 illustrates the output regulated voltage in the presence of a sinusoidal noise of 100radians/sec superimposed on the averaged output voltage. It can be observed that the adaptive system has decreased the noise in a factor of 6.

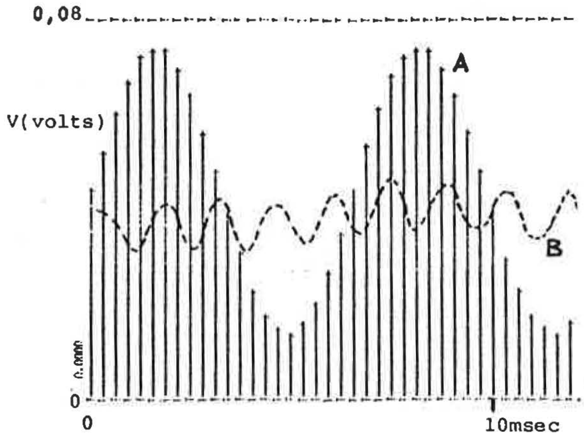

Fig. 12.- Output voltage with additive noise :
A) without adaptive control
B) with adaptive control

6.- State Observation

In sections 3 and 4 the inductor current has not been derived from the simulated converter model, but it has been obtained by means of a Luenberger observer [3]

The objective of the observer is adjusting the eigenvalues of the error equation e(t), so that the observer eigenvalues will be faster than those of the converter. The eigenvalaues of the converter modelled in section 2 are

$$\lambda_{1,2} = -32,25 \pm j\ 1451$$

chossing $\lambda^o_{1,2} = -64,5$ for the observer, we have obtained the results shown in figure 13.

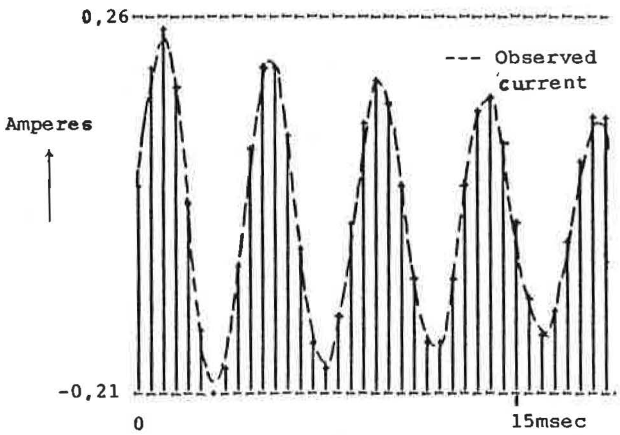

Fig. 13.- Comparison between inductor current derived from the converter model and that estimated by the observer

7.- Conclusions

Different control strategies for a buck switching regulator have been analyzed using the simulation program CSMP. First, a design based on the closed-loop pole assignment has been studied. This alternative acts directly on the system dynamics, but it needs at least one additional integrator to remove the steady-state offset which results after the introduction of an input perturbation.

Although the optimal linear regulator is an attractive way of design -as the deviations of v and i can be penalyzed separately- it is not easy to determine if desired penalty level represents a dynamic behaviour not attainable by the system. In the case of a buck regulator, the pole assignment is a better alternative. The OLR should be used in complex converters in which adjusting the eigenvalues separately is not straightforward. Finally, the adaptive control not only provides a similar dynamic behaviour to that obtained by means of an optimal pole assignment, but it also reduces significantly the additive noise in the output variables.

8.- References

[1] R.D. Middlebrook and S.Ćuk,"A general unified approach to modelling switching converter power stages," 1976 IEEE Power Electronics Specialist Conference Record. pp. 18-34

[2] F.E. Thau, "A feedback compensator design procedure for switching regulators" IEEE Transactions on Industrial Electronics and Control Instrumentation, vol. IECI-26, nº 2. May 1979 pp. 104-110

[3] J. D'Azzo and C.H. Houpis, "Linear Control System analysis and design" Mc. Graw-Hill 1981

[4] B.D.O. Anderson and J.B. Moose "Linear optimal control" Prentice Hall, 1971

[5] Y. Landau "Adaptive control : the model reference approach" Marcel Dekker Inc. 1979